ENGINEERING SCIENCES AND APPLIED MATHEMATICS

Degree Types: PhD, MS

The program in Engineering Science and Applied Mathematics (https://www.mccormick.northwestern.edu/applied-math/graduate/) educates students in mathematical methods and their application to various scientific and engineering fields.

Our program emphasizes techniques associated with asymptotic analysis, bifurcation theory, dynamical systems, numerical analysis, ordinary and partial differential equations, singular perturbations, stability theory, and stochastic processes. These techniques are employed to address problems arising in science and engineering including fluid mechanics, microbiology, neurobiology, ecology, geophysics, materials, diffusion processes, fiber optics, interfacial phenomena, pattern formation, solid mechanics, solidification theory, wave phenomena, and social sciences.

Students are given broad-based training encompassing mathematical methods and fields of science or engineering where significant applications of mathematics are made.

Additional resources:

- Department website (https://www.mccormick.northwestern.edu/applied-math/)
- Program handbook(s)

Degrees Offered

- Engineering Sciences and Applied Mathematics BS/MS (https://catalogs.northwestern.edu/tgs/engineering-sciences-applied-mathematics/engineering-sciences-applied-mathematics-bach-mast/)
- Engineering Sciences and Applied Mathematics MS (https://catalogs.northwestern.edu/tgs/engineering-sciences-applied-mathematics/engineering-sciences-applied-mathematics-ms/)
- Engineering Sciences and Applied Mathematics PhD (https://catalogs.northwestern.edu/tgs/engineering-sciences-applied-mathematics/engineering-sciences-applied-mathematics-phd/)

Engineering Sciences and Applied Math: MS

Learning objective(s)/Students should be able to…

- Develop skills in core applied mathematics skills.
- Develop expertise in a single area of mathematics.
- Develop expertise in an area of application of mathematics.

Engineering Sciences and Applied Math: PHD

Learning objective(s)/Students should be able to…

- Demonstrate mastery of core mathematical concepts.
- Design an original research project.
- Communicate research progress.
- Develop classroom skills.
- English language proficiency for international doctoral students.

Engineering Sciences and Applied Mathematics Courses

ES_APPM 311-0 Methods of Applied Mathematics (1 Unit)
Ordinary differential equations: review of elementary ODEs, initial and boundary value problems, Fredholm Alternative Theorem, Power series solution of ODEs, Special functions, Sturm-Liouville eigenvalue problems, Eigenfunction expansions, Fourier series. Partial Differential Equations: Classification, Heat, Wave and Laplace equations and their applications, Solution by separation of variables, Series solutions, Full and partial eigenfunction expansions.
Prerequisites: an elementary differential equations course, e.g., MATH 250-0 or GEN_ENG 205-4 or GEN_ENG 206-4.

ES_APPM 312-0 Complex Variables (1 Unit)
Imaginary numbers and complex variables, analytic functions, calculus of complex functions, contour integration with application to transform inversion, conformal mapping.
Prerequisite: GEN_ENG 205-4, GEN_ENG 206-4, or MATH 250-0.

ES_APPM 322-0 Applied Dynamical Systems (1 Unit)
Example-oriented survey of nonlinear dynamical systems, including chaos. Combines numerical exploration of differential equations describing physical problems with analytic methods and geometric concepts. Applications to mechanical, fluid dynamical, electrical, chemical, and biological systems.
Prerequisites: GEN_ENG 205-4, GEN_ENG 206-4, or MATH 250-0. ES_APPM 311-1 is recommended.

ES_APPM 344-0 High Performance Scientific Computing (1 Unit)
Solving partial differential equations using high performance computing platforms. Basic C programming. Distributed computing using MPI. GPU programming using CUDA. Adaptation of algorithms for solving PDE's to different architectures.

ES_APPM 345-0 Applied Linear Algebra (1 Unit)
Understanding and implementation of algorithms to calculate matrix decompositions such as eigenvalue/vector, LU, QR, and SVD decompositions. Applications include data-fitting, image analysis, and ranking algorithms.

ES_APPM 346-0 Modeling and Computation in Science & Engineering (1 Unit)
Advanced techniques for initial value problems, differential algebraic systems, bifurcations, chaos, and partial differential equations. Applications drawn from different physical areas.
Prerequisites: MATH 228-2, MATH 240-0, and MATH 250-0; or GEN_ENG 205-4 and PHYSICS 135-1, PHYSICS 135-2; or equivalent; familiarity with a programming language; or consent of instructor.

ES_APPM 370-1 Introduction to Computational Neuroscience (1 Unit)
From neurons to networks. Ion channels, Hodgkin-Huxley framework, simplified models, cable equation, synapses, spike triggered average, and optimal stimulus. Feedforward and recurrent firing rate networks. Statistical approach, Bayesian modeling. Brief introduction to numerical methods.

ES_APPM 372-0 Introduction to the Analysis of RNA Sequencing Data (1 Unit)
This course will give an introduction to the theory and practice of analyzing high-throughput RNA sequencing through lectures and hands-on exercises. The basic topics covered will include: 1) the format of/working with raw sequencing data; 2) aligning reads to a reference genome; 3) the format of/working with aligned SAM/BAM files; 4) different ways to perform read-based gene counting; 4) How to visually explore reads and read counts; 5) variance shrinkage and principal

components; 6) The theory of/doing differential expression analysis. Additional topics will be covered as time permits, based in part upon the interests of the course participants.

ES_APPM 375-1 Quantitative Biology I: Experiments, Data, Models, and Analysis (1 Unit)
High-resolution, high-throughput, and dynamic imaging and sequencing data is the substrate of modern biology. The course consists of case-studies where we learn how to computational work with, analyze, and make sense of experimental dataset using fundamental principles of mathematics, statistics, and physics. No formal course prerequisites. Programming in python.

ES_APPM 375-2 Quantitative Biology II: Experiments, Data, Models, and Analysis (1 Unit)
High-resolution, high-throughput, and dynamic imaging and sequencing data is the substrate of modern biology. In this course we learn how to perform experiments, and computational work with, analyze, and make sense of experimental dataset using fundamental principles of mathematics, statistics, and physics. No formal course prerequisites. Programming in python.

ES_APPM 395-0 Special Topics (1 Unit)

ES_APPM 398-0 Introduction to Applied Math Research (0 Unit)
This is a seminar course where ESAM faculty present their current and planned research topics in applied mathematics.

ES_APPM 401-0 Options Pricing: Theory and Applications (1 Unit)
Consideration of ordinary and elementary partial differential equations models of problems in science and engineering, arising in various areas of application.
Prerequisites: Permission of instructor and department.

ES_APPM 411-1 Differential Equations of Mathematical Physics (1 Unit)
Methods for solving linear, ordinary, and partial differential equations of mathematical physics. Green's functions, distribution theory,integral equations, transforms, potential theory, diffusion equation, wave equation, maximum principles, and variational methods.

ES_APPM 411-2 Differential Equations of Mathematical Physics (1 Unit)
Methods for solving linear, ordinary, and partial differential equations of mathematical physics. Green's functions, distribution theory,integral equations, transforms, potential theory, diffusion equation, wave equation, maximum principles, and variational methods.

ES_APPM 411-3 Differential Equations of Mathematical Physics (1 Unit)
Methods for solving linear, ordinary, and partial differential equations of mathematical physics. Green's functions, distribution theory,integral equations, transforms, potential theory, diffusion equation, wave equation, maximum principles, and variational methods.

ES_APPM 412-0 Methods of Nonlinear Analysis (1 Unit)
Methods for analyzing nonlinear problems in science and engineering. Constructive approach to bifurcation theory and stability theory, dynamical response of nonlinear systems, nonlinear oscillations and phase plane analysis, nonlinear wave propagation, and perturbation methods. Applications.

ES_APPM 420-1 Asymptotic and Perturbation Methods in Applied Mathematics (1 Unit)
Most mathematical models do not admit exact solutions. Asymptotic and perturbation methods provide powerful techniques for obtaining approximate solutions, which allow one to draw physical conclusions and guide numerical simulations of the model. This course covers such methods addressing the following topics: asymptotic expansions and series; approximate solution of algebraic and transcendental equations; asymptotic behavior of solutions of ODEs near ordinary and singular points; regular and singular problems for ODEs; matched asymptotic expansions, boundary layers; strained coordinates; multiple scales; Mathieu equation and Floquet theory; linear and nonlinear oscillators; equations with slowly varying coefficients; BVPs by multiple scales; Hopf bifurcation.

ES_APPM 420-2 Asymptotic and Perturbation Methods in Applied Mathematics (1 Unit)
Building on ES_APPM 420-1, this course covers advanced perturbation and asymptotic methods. They may include methods for ordinary differential equations (e.g. WKB method), for integrals (e.g. Laplace's method, stationary phase, steepest descents), and for partial differential equations (multiple scales, subcharacteristics, initial layers, homogenization).
Prerequisites: ES_APPM 420-1 or consent of instructor.

ES_APPM 420-3 Asymptotic and Perturbation Methods in Applied Mathematics (1 Unit)
Asymptotic expansions of integrals. Regular and singular perturbation methods for ordinary and partial differential equations. Boundary layer theory. Matched asymptotic expansions. Homogenization. Two-time and uniform expansions. Wave propagation and WKBJ method. Turning point theory. Nonlinear oscillations. Bifurcation and stability theory.

ES_APPM 421-1 Models in Applied Mathematics (1 Unit)
Applications to illustrate typical problems and methods of applied mathematics. Mathematical formulation of models for phenomena in science and engineering, problem solution, and interpretation of results. Examples from solid and fluid mechanics, combustion, diffusion phenomena, chemical and nuclear reactors, and biological processes.

ES_APPM 426-0 Theory of Flows With Small Inertia (1 Unit)
Asymptotic methods for flows with small inertia: flows past bodies and matching procedures. Slowly varying flows: lubrication theory and Hele-Shaw flow; swimming of microorganisms and suspension of particles.

ES_APPM 429-0 Hydrodynamic Stability Theory (1 Unit)
Mathematical theory of hydrodynamic states; energy methods, linear theories, and nonlinear bifurcation theories. Convective, centrifugal, and shear flow instabilities. Instability of unsteady flows and systems having interfaces. Physical mechanisms and results of experiments.

ES_APPM 430-0 Wave Propagation (1 Unit)
Problems of linear wave propagation; applications to acoustics, optics, electromagnetics, elasticity, and fluids. Radiation, transmission, scattering, diffraction, dispersion, layered media, wave-guides, coupled fluid solid waves, and inverse problems. Development and application of perturbation, asymptotic, numerical, and integral transform methods.

ES_APPM 440-0 Integral Equations & Applications (1 Unit)
Integral equations in various scientific theories and their relation to differential equations. Methods of solving linear problems with Hilbert-Schmidt, Cauchy, and Wiener-Hopf type kernels; applications. Nonlinear problems in bifurcation phenomena.

ES_APPM 442-0 Stochastic Differential Equations (1 Unit)
Brownian motion and Langevin's equation. Ito and Stratonovich stochastic integrals. Stochastic calculus and Ito's formula. SDEs and PDEs of Kolmogorov, Fokker-Planck, and Dynkin. Boundary conditions, exit times, exit distributions, stability. Asymptotic analysis of SDE, the Smoluchowski-Kramers approximation, and diffusion approximation to Markov chains. Applications.

ES_APPM 444-0 High Performance Scientific Computing (1 Unit)
Solving partial differential equations using high performance computing platforms. Basic C programming. Distributed computing using MPI. GPU

programming using CUDA. Adaptation of algorithms for solving PDE's to different architectures.

ES_APPM 445-0 Iterative Methods for Elliptic Equations (1 Unit)
Analysis and application of numerical methods for solving large systems of linear equations, which often represent the bottleneck when computing solutions to equations arising in fluid mechanics, elasticity, and other applications in the physical sciences. They also occur in many imaging, optimization, and machine learning applications. The class presents efficient solution methods, which are imperative in all of these applications. The methods include relaxation methods, gradient methods, including conjugate gradient and conjugate residual type methods with and without preconditioning, and multigrid methods.

ES_APPM 446-1 Numerical Solution of Partial Differential Equations (1 Unit)
Introductory course on using a range of finite-difference methods to solve initial-value and initial-boundary-value problems involving partial differential equations. The course covers theoretical aspects like the convergence and stability properties of such methods and the well-posedness of initial-value problems, as well as explicit implementation considerations. The course involves significant programming.

ES_APPM 446-2 Numerical Solution of Partial Differential Equations (1 Unit)
Course on using spectral methods to solve partial differential equations. We will cover the exponential convergence of spectral methods for periodic and non-periodic problem, and a general framework to efficiently solve nearly arbitrary PDEs with spectral methods. The course mixes discussion of theoretical aspects of spectral methods with their practical application using Python. The students will learn the algorithms underpinning the Dedalus spectral code, and how to use Dedalus.

ES_APPM 447-0 Boundary Integral Method (1 Unit)
Numerical solution of Fredholm and Volterra integral equations and of integro-differential equations. Convergence and stability of algorithms.

ES_APPM 448-0 Numerical Methods for Random Processes (1 Unit)
Analysis and implementation of numerical methods for random processes: random number generators, Monte Carlo methods, Markov chains, stochastic differential equations, and applications.

ES_APPM 449-0 Numerical Methods for Moving Interfaces (1 Unit)
methods for simulating sharp interfaces. Marker particle, level set, fast marching, volume of fluid, and phase fields methods.

ES_APPM 451-0 Mathematical Models in Biology (0.5 Unit)
This half-credit course discusses classical mathematical models of biological systems, with emphasis on the modeling process. Modeling tools used include ordinary and partial differential equations as well as agent-based frameworks. Topics may include chemotaxis, cellular aggregation, morphogenesis, and other classical systems that lend themselves to mathematical modeling. No biological background is required.

ES_APPM 472-0 Introduction to the Analysis of RNA Sequencing Data (1 Unit)
This course is an introduction to the theory and practice of analyzing high-throughput RNA sequencing data. This includes working with data up to and including a differential expression analysis, and troubleshooting issues. The course will also cover some of the theory, i.e., we will discuss the mathematical and statistical assumptions made in order to perform the various steps described above.

ES_APPM 475-0 What Do Your Data Say? A course to help you better understand your data. (1 Unit)
Modern data streams, whether from biomedical research labs, environmental research teams, or social-media survey projects, are increasingly quantitative and noisy. In this class, we will teach you to think quantitatively and statistically about your data, so that you can confidently answer the question "What do my data (actually) say?"

ES_APPM 479-0 Data Driven Methods for Dynamical Systems (1 Unit)
The course will survey methods for characterizing time-series data by reading and discussing primary literature and implementing and testing methods. Students will simulate time-series from deterministic, chaotic, and stochastic systems and apply a range of data-driven methods. The goal is to understand the suitability of different methods for characterizing dynamical systems with noise, nonlinearities, and other characteristics.

ES_APPM 495-0 Selected Topics in Applied Mathematics (0.5-1 Unit)
Topics selected from research of current interest in applied mathematics.

ES_APPM 499-0 Projects (1 Unit)
Special projects to be carried out under faculty direction. Permission of instructor and department required.

ES_APPM 519-0 Responsible Conduct of Research Training (0 Unit)

ES_APPM 590-0 Research (1-4 Units)
Independent investigation of selected problems pertaining to thesis or dissertation. May be repeated for credit.