

# MATHEMATICS

Degree Types: PhD

The Graduate Program in the Department of Mathematics (<https://www.math.northwestern.edu/graduate/>) aims to develop students into productive research mathematicians. The extremely low student-faculty ratio, approximately two-to-one, allows for close relationships between students and faculty.

Department research strengths include algebra, algebraic geometry, algebraic topology, classical and modern analysis, dynamical systems, mathematical physics, number theory, partial differential equations, probability and representation theory.

The department maintains close ties with the larger community of mathematicians at Northwestern University.

## Additional resources:

- Department website (<https://www.math.northwestern.edu/graduate/>)
- Program handbook(s)

## Degree Offered

- Mathematics PhD (<https://catalogs.northwestern.edu/tgs/mathematics/mathematics-phd/>)

Learning objective(s)/Students should be able to...

- Design and implement a research project
- Write a research paper
- Articulate broader impacts of research
- Create and deliver seminar and conference presentations
- Develop pedagogical skills in the classroom
- Search for and apply for jobs

## Mathematics Courses

### MATH 300-0 Foundations of Higher Mathematics (1 Unit)

Introduction to fundamental mathematical structures, including sets, functions, equivalence relations, and cardinal numbers. Elementary logic and proof techniques. Students may not receive credit for MATH 300-0 after passing any of MATH 320-1, MATH 321-1, MATH 330-1, or MATH 331-1.

Prerequisite: MATH 240-0 or MATH 281-3 or MATH 285-1 or MATH 290-1 or MATH 291-1 or GEN\_ENG 205-1 or GEN\_ENG 206-1 or consent of the department.

*Formal Studies Distro Area*

### MATH 300-BR Foundations of Higher Mathematics (1 Unit)

For participants in the Causeway Postbaccalaureate Program only. Introduction to fundamental mathematical structures, including sets, functions, equivalence relations, and cardinal numbers. Elementary logic and proof techniques. Additional topics selected by the instructor.

### MATH 306-0 Combinatorics & Discrete Mathematics (1 Unit)

Discrete mathematics, inductive reasoning, counting problems, binomial coefficients and Pascal's triangle, Fibonacci numbers, set and integer partitions, and generating functions.

Prerequisite: MATH 240-0 or MATH 281-3 or MATH 285-1 or MATH 290-1 or MATH 291-1 or GEN\_ENG 205-1 or GEN\_ENG 206-1.

*Formal Studies Distro Area*

### MATH 308-0 Graph Theory (1 Unit)

Introduction to graph theory: graphs, trees, matchings, planar graphs, and colorings. Additional topics as time permits.

Prerequisite: MATH 291-1 or MATH 300-0 or MATH 306-0.

*Formal Studies Distro Area*

### MATH 310-1 Probability and Stochastic Processes (1 Unit)

Axioms of probability. Conditional probability and independence. Random variables. Joint distributions. Expectation. Limit theorems: the weak law of large numbers and the central limit theorem. Students may not receive credit for both MATH 310-1 and any of MATH 311-1, MATH 314-0, MATH 385-0, STAT 320-1, STAT 383-0, IEMS 202-0, or ELEC\_ENG 302-0.

Prerequisite or corequisite: MATH 226-0 or MATH 281-2; and MATH 228-2 or MATH 230-2 or MATH 234-0 (former), or MATH 281-2 or MATH 285-3 or MATH 290-3 or MATH 291-3 or ES\_APPM 252-2.

*Formal Studies Distro Area*

### MATH 310-2 Probability and Stochastic Processes (1 Unit)

Discrete-time Markov chains, recurrence and transience. Students may not receive credit for both MATH 310-2 and MATH 311-2.

Prerequisites: MATH 240-0 or MATH 281-3 or MATH 285-1 or MATH 290-1 or MATH 291-1 or GEN\_ENG 205-1 or GEN\_ENG 206-1; and MATH 310-1 or MATH 311-1 or MATH 314-0 or MATH 385-0 or STAT 320-1 or STAT 383-0 or IEMS 202-0 or ELEC\_ENG 302-0.

*Formal Studies Distro Area*

### MATH 310-3 Probability and Stochastic Processes (1 Unit)

Continuous-time Markov chains, queues, population growth models. Brownian motion and other diffusion processes. Additional topics as time permits. Students may not receive credit for both MATH 310-3 and MATH 311-3.

Prerequisite: MATH 310-2 or MATH 311-2.

*Formal Studies Distro Area*

### MATH 311-1 MENU: Probability and Stochastic Processes (1 Unit)

Probability spaces. Random variables. Independence. Distributions. Generating functions. The central limit theorem. Students may not receive credit for both MATH 311-1 and any of MATH 310-1, MATH 314-0, MATH 385-0, STAT 320-1, STAT 383-0, IEMS 202-0, or ELEC\_ENG 302-0.

Prerequisite: MATH 226-0 or MATH 281-2; and MATH 291-3, or MATH 300-0 and any one of MATH 290-3, MATH 281-2, MATH 285-3 or ES\_APPM 252-2; or consent of the department. Recommended: MATH 320-1 or MATH 321-1.

*Formal Studies Distro Area*

### MATH 311-2 MENU: Probability and Stochastic Processes (1 Unit)

Markov chains, convergence of random variables, random processes, renewals, and queues. Students may not receive credit for both MATH 311-2 and MATH 310-2.

Prerequisite: MATH 311-1 or consent of the department.

*Formal Studies Distro Area*

### MATH 311-3 MENU: Probability and Stochastic Processes (1 Unit)

Stationary processes, martingales, and diffusion processes. Students may not receive credit for both MATH 311-3 and MATH 310-3.

Prerequisite: MATH 311-2 or consent of the department.

*Formal Studies Distro Area*

### MATH 314-0 Probability and Statistics for Econometrics (1 Unit)

Introduction to probability theory and statistical methods, including properties of probability distributions, sampling distributions, estimation, confidence intervals and hypothesis testing. For students planning to take ECON 381-1. Students may not receive credit for both MATH 314-0 and any of MATH 310-1, MATH 311-1, MATH 385-0, STAT 320-1, STAT 383-0, IEMS 202-0, or ELEC\_ENG 302-0. Prerequisite or corequisite: MATH 226-0 or MATH 281-2; and MATH 228-2 or MATH 230-2 or

MATH 234-0 (former) or MATH 281-1 or MATH 285-3 or MATH 290-3 or MATH 291-3 or ES\_APPM 252-2.

*Formal Studies Distro Area*

**MATH 320-1 Real Analysis (1 Unit)**

Analysis on the real line: axiomatic development of the real number system, sequences and series of real numbers, continuity, and differentiability. Students may not receive credit for both MATH 320-1 and MATH 321-1.

Prerequisite: MATH 226-0 or MATH 281-2; and MATH 300-0 or MATH 291-3; or consent of the department.

*Formal Studies Distro Area*

**MATH 320-2 Real Analysis (1 Unit)**

Analysis on the real line: the Riemann integral and sequences and series of functions. Additional topics as time permits. Students may not receive credit for both MATH 320-2 and MATH 321-2.

Prerequisite: MATH 320-1 or MATH 321-1.

*Formal Studies Distro Area*

**MATH 320-3 Real Analysis (1 Unit)**

Analysis on Euclidean spaces: the topology of Euclidean spaces, limits, continuity, and differentiability, including the inverse and implicit function theorems. Additional topics as time permits. Students may not receive credit for both MATH 320-3 and MATH 321-2.

Prerequisite: MATH 320-2.

*Formal Studies Distro Area*

**MATH 321-1 MENU: Real Analysis (1 Unit)**

Analysis on metric spaces: the real number system, the topology of metric spaces, sequences and series, continuity, and differentiability. Students may not receive credit for both MATH 321-1 and MATH 320-1.

Prerequisite: consent of the department.

*Formal Studies Distro Area*

**MATH 321-2 MENU: Real Analysis (1 Unit)**

Analysis on metric spaces: the Riemann integral, sequences and series of functions, and functions of several variables, including the inverse and implicit function theorems. Students may not receive credit for both MATH 321-2 and either MATH 320-2 or MATH 320-3.

Prerequisite: MATH 321-1.

*Formal Studies Distro Area*

**MATH 321-3 MENU: Real Analysis (1 Unit)**

Lebesgue measure and the Lebesgue integral. Additional topics as time permits.

Prerequisite: MATH 321-2.

*Formal Studies Distro Area*

**MATH 325-0 Complex Analysis (1 Unit)**

Complex numbers. Analytic functions. Cauchy's theorem and the Cauchy integral formula. Series. Residues. Students may not receive credit for both MATH 325-0 and either MATH 382-0 or ES\_APPM 312-0.

Prerequisites: MATH 226-0 or MATH 281-2; and MATH 228-2 or MATH 230-2 or MATH 234-0 (former) or MATH 281-2 or MATH 285-3 or MATH 290-3 or MATH 291-3 or ES\_APPM 252-2; and MATH 240-0 or MATH 281-3 or MATH 285-1 or MATH 290-1 or MATH 291-1 or GEN\_ENG 205-1 or GEN\_ENG 206-1.

*Formal Studies Distro Area*

**MATH 327-0 Mechanics for Mathematicians (1 Unit)**

Fundamental mathematical ideas arising in classical mechanics: Newtonian mechanics, Lagrangian formalism and the calculus of variations, motion with constraints, symmetries and conservation laws, Hamiltonian mechanics, and Liouville's theorem. No prior knowledge of physics required. Students may not receive credit for MATH 327-0 after taking PHYSICS 330-1.

Prerequisites: MATH 226-0 or MATH 281-3; and MATH 228-2 or MATH 230-2 or MATH 234-0 (former) or MATH 281-2 or MATH 285-3 or MATH 290-3 or MATH 291-3 or ES\_APPM 252-2; and MATH 240-0 or MATH 281-3 or MATH 285-1 or MATH 290-1 or MATH 291-1 or GEN\_ENG 205-1 or GEN\_ENG 206-1.

*Formal Studies Distro Area Interdisciplinary Distro - See Rules*

(<https://catalogs.northwestern.edu/undergraduate/arts-sciences/#schoolrequirementstext>) *Natural Sciences Distro Area*

**MATH 330-1 Abstract Algebra (1 Unit)**

Group theory. Students may not receive credit for both MATH 330-1 and MATH 331-1.

Prerequisite: MATH 291-1 or MATH 300-0.

*Formal Studies Distro Area*

**MATH 330-2 Abstract Algebra (1 Unit)**

Ring theory, including polynomial rings. Students may not receive credit for both MATH 330-2 and MATH 331-2.

Prerequisite: MATH 330-1 or MATH 331-1.

*Formal Studies Distro Area*

**MATH 330-3 Abstract Algebra (1 Unit)**

Field theory and Galois theory. Students may not receive credit for both MATH 330-3 and MATH 331-3.

Prerequisite: MATH 330-2 or MATH 331-2.

*Formal Studies Distro Area*

**MATH 331-1 MENU: Abstract Algebra (1 Unit)**

Group theory, including the Sylow theorems. Students may not receive credit for both MATH 331-1 and MATH 330-1.

Prerequisite: consent of the department.

*Formal Studies Distro Area*

**MATH 331-2 MENU: Abstract Algebra (1 Unit)**

Ring theory, including polynomial rings. Module theory, including canonical forms of operators on vector spaces. Students may not receive credit for both MATH 331-2 and MATH 330-2.

Prerequisite: MATH 331-1.

*Formal Studies Distro Area*

**MATH 331-3 MENU: Abstract Algebra (1 Unit)**

Field theory and Galois theory. Students may not receive credit for both MATH 331-3 and MATH 330-3.

Prerequisite: MATH 331-2.

*Formal Studies Distro Area*

**MATH 334-0 Linear Algebra: Second Course (1 Unit)**

Vector spaces. Linear maps. Eigenvalues, eigenvectors and invariant subspaces. Inner product spaces. Canonical forms of operators on real and complex vector spaces.

Prerequisite: MATH 300-0 or MATH 291-2.

*Formal Studies Distro Area*

**MATH 336-1 Introduction to the Theory of Numbers (1 Unit)**

Divisibility and prime numbers. Congruences. Quadratic reciprocity. Diophantine equations.

Prerequisite: MATH 228-1 or MATH 230-1 or MATH 281-1 or MATH 285-2 or MATH 290-2 or MATH 291-2 or ES\_APPM 252-1.

*Formal Studies Distro Area*

**MATH 336-2 Introduction to the Theory of Numbers (1 Unit)**

Topics in analytic and algebraic number theory.

Prerequisite: MATH 336-1.

*Formal Studies Distro Area*

**MATH 340-0 Geometry (1 Unit)**

Axioms for Euclidean geometry. Non-Euclidean geometry. Projective geometry. Introduction of coordinate systems from the axioms. Quadrics. Erlangen program. Introduction to plane algebraic curves.

Prerequisite: MATH 300-0 or MATH 291-1.

*Formal Studies Distro Area*

#### **MATH 342-0 Introduction to Differential Geometry (1 Unit)**

Differential geometry of curves and surfaces in three-dimensional space: curves, regular surfaces, the Gauss map, and additional topics as time permits.

Prerequisites: MATH 226-0 or MATH 281-2; and MATH 228-2 or MATH 230-2 or MATH 234-0 (former) or MATH 281-2 or MATH 285-3 or MATH 290-3 or MATH 291-3 or ES\_APPM 252-2; and MATH 240-0 or MATH 281-3 or MATH 285-1 or MATH 290-1 or MATH 291-1 or GEN\_ENG 205-1 or GEN\_ENG 206-1.

*Formal Studies Distro Area*

#### **MATH 344-1 Introduction to Topology (1 Unit)**

Topological spaces, continuity, connectedness, compactness, countability and separation axioms.

Prerequisite: MATH 320-1 or MATH 321-1.

*Formal Studies Distro Area*

#### **MATH 344-2 Introduction to Topology (1 Unit)**

The fundamental group. Classification of covering spaces. Additional topics as permits.

Prerequisites: MATH 344-1, and either MATH 330-1 or MATH 331-1.

*Formal Studies Distro Area*

#### **MATH 351-0 Fourier Analysis and Boundary Value Problems (1 Unit)**

Fourier series with applications to partial differential equations arising in physics and engineering. Students may not receive credit for both MATH 351-0 and any of MATH 381-0, MATH 360-2, or ES\_APPM 311-2.

Prerequisite: MATH 250-0 or MATH 281-3 or MATH 360-1 or GEN\_ENG 206-4 or GEN\_ENG 206-4.

*Formal Studies Distro Area*

#### **MATH 353-0 Qualitative Theory of Differential Equations (1 Unit)**

Qualitative theory of ordinary differential equations: linear systems, phase portraits, periodic solutions, stability theory, Lyapunov functions, and chaos. Students may not receive credit for both MATH 353-0 and MATH 360-2.

Prerequisite: MATH 250-0 or MATH 281-3 or MATH 360-1 or GEN\_ENG 205-4 or GEN\_ENG 206-4.

*Formal Studies Distro Area*

#### **MATH 360-1 MENU: Applied Analysis (1 Unit)**

Linear ordinary differential equations, systems of linear ordinary differential equations, and applications. Students may not receive credit for both MATH 360-1 and any of MATH 250-0, MATH 281-3, GEN\_ENG 205-4, GEN\_ENG 206-4.

Prerequisite: MATH 226-0 or MATH 281-2; and MATH 290-3 or MATH 291-3.

*Formal Studies Distro Area*

#### **MATH 360-2 MENU: Applied Analysis (1 Unit)**

Qualitative analysis of systems of ordinary differential equations. Linear partial differential equations. Fourier series and orthogonal functions. Applications. Students may not receive credit for both MATH 360-2 and any of MATH 381-0, MATH 351-0, or ES\_APPM 311-2.

Prerequisite: MATH 360-1.

*Formal Studies Distro Area*

#### **MATH 368-0 Introduction to Optimization (1 Unit)**

Methods and concepts of optimization theory: linear programming, duality, convexity, and Kuhn-Tucker theory.

Prerequisites: MATH 226-0 or MATH 281-2; and MATH 291-3, or MATH 300-0 and one of MATH 228-2, MATH 230-2, MATH 234-0 (former), MATH 281-2, MATH 285-3, MATH 290-3, or ES\_APPM 252-2.

*Formal Studies Distro Area*

#### **MATH 370-0 Mathematical Logic (1 Unit)**

Mathematical formulation and rigorous discussion of logical systems, particularly the propositional calculus and the functional calculi of first and second order. Well-formed formulae, formal languages, proofs, tautologies, effective procedures, deduction theorems, axiom schemata.

Prerequisite: MATH 300-0 or MATH 291-3 or consent of the instructor.

*Formal Studies Distro Area*

#### **MATH 410-1 Analysis (1 Unit)**

Real analysis. Topological spaces, metric spaces. Lebesgue measure and integration. Function spaces, including Banach and Hilbert spaces. Elementary functional analysis. Weak convergence.

#### **MATH 410-2 Analysis (1 Unit)**

Real analysis. Topological spaces, metric spaces. Lebesgue measure and integration. Function spaces, including Banach and Hilbert spaces. Elementary functional analysis. Weak convergence.

#### **MATH 410-3 Introduction to Modern Analysis (1 Unit)**

Complex analysis. Holomorphic functions, Cauchy's theorem, power series, harmonic functions, conformal mapping, analytic continuation.

#### **MATH 413-1 Functions of a Complex Variable (1 Unit)**

Holomorphic functions: theorems of Cauchy, Morera, and Rouché residue and open mapping theorems; harmonic and entire functions; analytic continuation; conformal mapping. Schlicht functions, functions of several complex variables, Hp spaces, and complex manifolds.

#### **MATH 414-0 Abstract Riemann Surfaces (1 Unit)**

Abstract Riemann Surfaces, differential forms, Poincare-Hopf formula, algebraic curves Riemann-Hurwitz formula, Riemann-Roch formula and applications, Jacobi variety and Abel theorem, and Uniformization theorem.

#### **MATH 415-1 Functional Analysis (1 Unit)**

Topological groups and topological vector spaces; Banach spaces, linear functionals, and operators; applications to functional equations.

#### **MATH 415-2 Functional Analysis (1 Unit)**

Topological groups and topological vector spaces; Banach spaces, linear functionals, and operators; applications to functional equations.

#### **MATH 420-1 Partial Differential Equations (1 Unit)**

Introduction to basic differential equations, with emphasis on the theory of partial differential equations.

Prerequisites: Advanced calculus and linear algebra or permission of instructor.

#### **MATH 420-2 Partial Differential Equations (1 Unit)**

Introduction to basic differential equations, with emphasis on the theory of partial differential equations.

Prerequisites: Advanced calculus and linear algebra or permission of instructor.

#### **MATH 420-3 Partial Differential Equations (1 Unit)**

Introduction to basic differential equations, with emphasis on the theory of partial differential equations.

Prerequisites: Advanced calculus and linear algebra or permission of instructor.

#### **MATH 425-1 Partial Differential Equations II (1 Unit)**

Nonlinear elliptic differential equations, nonlinear hyperbolic differential equations, pseudodifferential operators, and other topics.

#### **MATH 425-2 Partial Differential Equations II (1 Unit)**

Nonlinear elliptic differential equations, nonlinear hyperbolic differential equations, pseudodifferential operators, and other topics.

**MATH 425-3 Partial Differential Equations II (1 Unit)**

Nonlinear elliptic differential equations, nonlinear hyperbolic differential equations, pseudodifferential operators, and other topics.

**MATH 428-0 Geometric Measure Theory & Applications (1 Unit)**

General measure theory, Hausdorff measure, area and co-area formulas, Sobolev functions, BV functions and set of finite perimeter, Gauss-Green theorem, differentiability and approximation, applications.

**MATH 429-0 Fourier Analysis (1 Unit)**

A short overview of classical Fourier analysis on the circle. Selected topics about Fourier analysis on the line and in Euclidean space.

Prerequisite: Permission of instructor.

**MATH 430-1 Dynamical Systems (1 Unit)**

Qualitative theory of differentiable dynamical systems, emphasizing global properties such as structural stability theorems.

**MATH 430-2 Dynamical Systems (1 Unit)**

Qualitative theory of differentiable dynamical systems, emphasizing global properties such as structural stability theorems.

**MATH 430-3 Dynamical Systems (1 Unit)**

Qualitative theory of differentiable dynamical systems, emphasizing global properties such as structural stability theorems.

**MATH 435-0 Ergodic Theory (1 Unit)**

Introduction to abstract ergodic theory, focusing on the asymptotic behavior of measure preserving transformations. Topics to be covered include: measure preserving transformations and flows, convergence theorems, recurrence properties, isomorphism invariants, and applications to problems in number theory, probability, and combinatorics.

Prerequisite: MATH 410-1.

**MATH 438-3 Interdisciplinary Nonlinear Dynamics (1 Unit)**

First quarter: Example-oriented survey of nonlinear dynamical systems, including chaos, combining numerical, analytical and geometrical approaches to differential chaos, combining numerical, analytical and geometrical approaches to differential equations. Second and third quarters: Interdisciplinary theoretical, computational and experimental projects involving complex systems in science and engineering directed by cross-disciplinary faculty teams.

**MATH 440-1 Geometry and Topology (1 Unit)**

Differentiable topology: differentiable manifolds; implicit function theorem and Sard's theorem; smooth vector bundles, tangent vectors, tensors, vector fields and flows. Lie derivatives, Lie groups and Lie algebras. Integral manifolds, Frobenius's theorem. Differential forms and the de Rham complex. Orientation, integration, Riemannian metrics, geodesics, exponential map.

**MATH 440-2 Geometry and Topology (1 Unit)**

Algebraic topology: The fundamental group of a space, covering spaces, and the Van-Kampen theorem. Singular homology, Mayer-Vietoris, degree and Euler characteristic.

**MATH 440-3 Geometry and Topology (1 Unit)**

de Rham cohomology, Mayer-Vietoris, Poincaré duality, singular homology and cohomology. Cohomology of cell complexes, simplicial cohomology, Čech cohomology. Cup product; sheaves.

Prerequisite: MATH 440-2.

**MATH 444-0 Hamiltonian Dynamics and Symplectic Geometry (1 Unit)**

Symplectic structure and cotangent bundle. Hamiltonian flow and their invariants. Integrable systems and stability. Lagrangian intersection

theory and symplectic fixed points theorems. Arnold conjecture on  $n$ -torus.

**MATH 445-1 Differential Geometry (1 Unit)**

Riemannian geometry: connections, geodesics, completeness, Jacobi fields, exponential map, constant curvature.

**MATH 445-2 Differential Geometry (1 Unit)**

Hodge theory: connections, curvature, de Rham complex, Hodge decomposition, Kähler manifolds, Chern-Weil theorem.

**MATH 445-3 Differential Geometry (1 Unit)**

Further topics: connections and curvature on principal and associated bundles; symplectic geometry, classical mechanics and geometric quantization; Dirac operators and index theorems.

**MATH 450-1 Probability Theory & Stochastic Analysis (1 Unit)**

Probability spaces, random variables, distribution functions, conditional probability, laws of large numbers, and central limit theorem. Random walk, Markov chains, martingales, and stochastic processes.

**MATH 450-2 Probability Theory and Stochastic Analysis (1 Unit)**

Random walk, Markov chains, martingales, and stochastic processes. Definition and properties of standard Brownian motion.

**MATH 450-3 Probability Theory and Stochastic Analysis (1 Unit)**

Stochastic Integration and stochastic differential calculus, with applications to diffusion processes.

**MATH 460-1 Algebraic Topology (1 Unit)**

Fundamental group and covering spaces.

**MATH 460-2 Algebraic Topology (1 Unit)**

Simplicial, singular, and cellular (co-) homology; universal coefficient and Künneth theorems.

Prerequisite: MATH 460-1.

**MATH 460-3 Algebraic Topology (1 Unit)**

Cohomology rings and Poincaré duality; Thom Isomorphism and characteristic classes.

Prerequisite: MATH 460-2.

**MATH 465-1 Algebraic Topology II (1 Unit)**

Cohomology theories and operations, homotopy and obstruction theory, and CW complexes; spectral sequences. Multiple registrations allowed.

**MATH 465-2 Algebraic Topology II (1 Unit)**

Cohomology theories and operations, homotopy and obstruction theory, and CW complexes; spectral sequences. Multiple registrations allowed.

**MATH 465-3 Algebraic Topology II (1 Unit)**

Cohomology theories and operations, homotopy and obstruction theory, and CW complexes; spectral sequences. Multiple registrations allowed.

**MATH 468-0 Homological Algebra (1 Unit)**

Exact sequences, Ext and Tor, and homological dimensions.

**MATH 470-1 Algebra (1 Unit)**

Free, permutation, solvable, simple, and linear groups. Actions of groups on sets; Sylow theorems. Rings and modules: polynomials and power series, Euclidean domains, PIDs, UFDs, and free and projective modules. Field and Galois theory. Extensions: algebraic, transcendental, normal, and integral. Splitting fields. Wedderburn theory. Commutative algebra: prime ideals; localization.

**MATH 470-2 Algebra (1 Unit)**

Extensions: algebraic, transcendental, normal, and integral. Splitting fields. Wedderburn theory.

**MATH 470-3 Algebra (1 Unit)**



Commutative algebra: prime ideals; localization. Homological algebra: linear algebra, abelian categories, complexes and homology, projective and injective resolutions, homotopies.

**MATH 477-0 Commutative Algebra (1 Unit)**

Research in commutative algebra: theory of depth (regular sequences, Koszul complexes), dimension theory, completions, Hilbert functions, Cohen-Macaulay modules, excellent rings, Hensel rings, and minimal resolutions.

Prerequisites: MATH 470-1, MATH 470-2, MATH 470-3 or equivalent.

**MATH 478-0 Representation Theory (1 Unit)**

Topics in the representation theory and cohomology of finite and infinite groups, including compact and non-compact Lie groups.

**MATH 482-1 Algebraic Number Theory (1 Unit)**

The theory of global and local fields; various special topics. 2. Abelian Galois extensions of algebraic number fields (class field theory). Complex multiplication, other examples, and relations with geometry.

**MATH 482-2 Algebraic Number Theory (1 Unit)**

Abelian Galois extensions of algebraic number fields (class field theory). Complex multiplication, other examples, and relations with geometry.

**MATH 483-1 Algebraic Geometry (1 Unit)**

Introduction to classical and scheme theoretic methods of algebraic geometry. Algebraic vector bundles, sheaf cohomology, the Riemann-Roch theorem for curves, and intersection theory.

**MATH 483-2 Algebraic Geometry (1 Unit)**

Introduction to classical and scheme theoretic methods of algebraic geometry. Algebraic vector bundles, sheaf cohomology, the Riemann-Roch theorem for curves, and intersection theory.

**MATH 483-3 Algebraic Geometry (1 Unit)**

Introduction to classical and scheme theoretic methods of algebraic geometry. Algebraic vector bundles, sheaf cohomology, the Riemann-Roch theorem for curves, and intersection theory.

**MATH 484-0 Lie Theory (1 Unit)**

Topics in the theory of Lie algebras and Lie groups including classification.

**MATH 485-1 Modular Forms (1 Unit)**

Introduction to the theory of modular forms. Congruence subgroups of  $SL(2, \mathbb{Z})$ , the definitions of modular functions and modular forms, Fourier expansions, Hecke operators, theta functions, modular curves.

**MATH 486-1 Algebraic K-Theory (1 Unit)**

Classical algebraic K-theory. Functors  $K_0$  and  $K_1$ ; origins in and relations with topology; congruence subgroup problem; techniques of computation: exact sequences, localization, resolution, and devissage; polynomial and related extensions; higher K-theories: Karoubi-Villamayor, Quillen.

**MATH 486-2 Algebraic K-Theory (1 Unit)**

Classical algebraic K-theory. Functors  $K_0$  and  $K_1$ ; origins in and relations with topology; congruence subgroup problem; techniques of computation: exact sequences, localization, resolution, and devissage; polynomial and related extensions; higher K-theories: Karoubi-Villamayor, Quillen.

**MATH 486-3 Algebraic K-Theory (1 Unit)**

Classical algebraic K-theory. Functors  $K_0$  and  $K_1$ ; origins in and relations with topology; congruence subgroup problem; techniques of computation: exact sequences, localization, resolution, and devissage; polynomial and related extensions; higher K-theories: Karoubi-Villamayor, Quillen.

**MATH 495-0 Statistical Phenomena in the Theory of Networks (1 Unit)**

This interdisciplinary course combines graph theory and probability theory to develop a rigorous foundation for the study of network-related problems.

**MATH 499-0 Independent Study (1 Unit)**

Permission of instructor and department required. May be repeated for credit.

**MATH 511-1 Topics in Analysis (1 Unit)**

Topics in Analysis and Probability Theory.

**MATH 511-2 Topics in Analysis (1 Unit)**

Topics in Analysis and Probability Theory.

**MATH 511-3 Topics in Analysis (1 Unit)**

Topics in Analysis and Probability Theory.

**MATH 512-1 Topics in Partial Differential Equations (1 Unit)**

Topics in Partial Differential Equations.

**MATH 512-2 Topics in Partial Differential Equations (1 Unit)**

Topics in Partial Differential Equations.

**MATH 512-3 Topics in Partial Differential Equations (1 Unit)**

Topics in Partial Differential Equations.

**MATH 513-1 Topics in Dynamical Systems (1 Unit)**

Topics in Partial Differential Equations.

**MATH 513-2 Topics in Dynamical Systems (1 Unit)**

Topics in Partial Differential Equations.

**MATH 513-3 Topics in Dynamical Systems (1 Unit)**

Topics in Partial Differential Equations.

**MATH 514-1 Topics in Geometry (1 Unit)**

Topics in Geometry.

**MATH 514-2 Topics in Geometry (1 Unit)**

Topics in Geometry.

**MATH 514-3 Topics in Geometry (1 Unit)**

Topics in Geometry.

**MATH 515-1 Topics in Geometry and Topology (1 Unit)**

This is a working seminar for students with interests in geometry, topology, and related fields. Its primary aim is to introduce students to research subjects of current interest to faculty members in these areas.

**MATH 515-2 Topics in Geometry and Topology (1 Unit)**

This is a working seminar for students with interests in geometry, topology, and related fields. Its primary aim is to introduce students to research subjects of current interest to faculty members in these areas.

**MATH 515-3 Topics in Geometry and Topology (1 Unit)**

This is a working seminar for students with interests in geometry, topology, and related fields. Its primary aim is to introduce students to research subjects of current interest to faculty members in these areas.

**MATH 516-1 Topics in Topology (1 Unit)**

Topics in Topology.

**MATH 516-2 Topics in Topology (1 Unit)**

This course is meant as a follow-up course to the MATH\_440-1,2,3 Geometry & Topology sequence. We will cover fundamental results and tools for modern algebraic topology such as (co)fibrations, obstruction theory, generalized cohomology theories, and spectral sequences.

**MATH 517-1 Topics in Algebra (1 Unit)**

Topics in Algebra.

**MATH 517-2 Topics in Algebra (1 Unit)**

Topics in Algebra.

**MATH 517-3 Topics in Algebra (1 Unit)**

Topics in Algebra.

**MATH 518-1 Topics in Number Theory (1 Unit)**

Topics in Number Theory.

**MATH 518-2 Topics in Number Theory (1 Unit)**

Topics in Number Theory.

**MATH 518-3 Topics in Number Theory (1 Unit)**

Topics in Number Theory.

**MATH 519-0 Responsible Conduct of Research Training (0 Unit)**

Responsible Conduct of Research Training.

**MATH 520-1 Topics in Mathematical Physics (1 Unit)**

Topics in Mathematical Physics.

**MATH 520-2 Topics in Mathematical Physics (1 Unit)**

Topics in Mathematical Physics.

**MATH 520-3 Topics in Mathematical Physics (1 Unit)**

Phase transitions are a central theme of statistical physics. In this course, we will discuss a canonical probabilistic framework, known as spin systems, for understanding phase transitions. These are large random systems of interacting spins assigned to the vertices of some underlying graph; well-known examples include percolation, the Ising and Potts models, and spin glasses. In this course, we will focus on understanding the rich phenomenology of the phase transitions spin systems undergo on integer lattices in dimensions two and higher, focusing primarily on percolation and the Ising model.

**MATH 521-1 Topics in Representation Theory (1 Unit)**

Topics Representation Theory.

**MATH 521-2 Topics in Representation Theory (1 Unit)**

Topics Representation Theory.

**MATH 521-3 Topics in Representation Theory (1 Unit)**

Topics Representation Theory.

**MATH 580-0 Seminar in College Teaching (0 Unit)**

A weekly two-hour seminar introducing the technique, philosophy, and practice of teaching undergraduate mathematics. Student presentations are critiqued by fellow students, as well as a senior faculty member.

**MATH 590-0 Research (1-3 Units)**

SEE DEPT FOR SECTION AND PERMISSION NUMBERS - Independent investigation of selected problems pertaining to thesis or dissertation. May be repeated for credit.